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# Plasmon interactions in the quark-gluon plasma

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**Abstract**

Yang-Mills theory at finite temperature is rewritten as a theory of plasmons which provides a Hamiltonian framework for perturbation theory with resummation of hard thermal loops.

# 1 Introduction

It is by now a well-recognized fact that the so-called hard thermal loops (HTL's) play a very important role in Yang-Mills theories at high temperatures [1]. Hard thermal loops and the corresponding effective action have been studied rather extensively over the last few years [2, 3, 4, 5]. Hard thermal loops describe Debye screening, Landau damping, etc., and a resummation of perturbation theory including their effects is essential to avoiding a class of infrared divergences due to interactions of the electrostatic type. While this is clear conceptually, the actual implementation of a resummed perturbation theory has not been easy [6, 7]. Further, even in a resummed perturbation theory, there are still infrared problems which can be cured only by incorporating magnetic screening effects as well [8]. Such effects are expected to be of order $g^2T$ where $g$ is the coupling constant and $T$ is the temperature. In order to analyze magnetic screening effects, one must first incorporate hard thermal loop effects (which are of order $gT$ ) and obtain an effective theory valid at lower scales, of order $g^2T$. There have been many related approaches to this question in recent literature, based on kinetic equations, thermal Feynman diagrams, etc. [9, 10, 11]. In this connection, it is worth emphasizing that magnetic screening involves the wave properties of the gauge fields. The magnetic mass may be considered as the (dynamically generated) mass gap of the effective three-dimensional theory of the zero Matsubara frequency modes; in terms of counting of powers of $\hbar$ this would seem to be a classical effect. However, the relevant classical theory is a wave theory, not a particle theory. In particular it is easy to see that, in the low energy limit where the zero Matsubara frequency approximation is valid, the single-particle wavefunctions have significant overlap, similar to what happens in Bose-Einstein condensation. Classical particle descriptions or corresponding kinetic equations will not suffice to generate a magnetic mass. One would thus like to have a way of incorporating the HTL-effects within a formalism which can systematically treat dynamical correlation effects as well as thermal corrections and preferably quantum corrections as well. The natural candidate for this would be an action formalism where one can add and subtract electric and magnetic mass terms and develop a resummed perturbation theory. While this is systematic, despite calculational complexity, the effective action for hard thermal loops involves nonlocality in time and some conceptual issues are better addressed in a Hamiltonian approach. Since the theory at finite temperature lacks manifest covariance anyway, there seems to be no serious drawback to a Hamiltonian analysis. In this paper we set up the basic framework of a resummed Hamiltonian analysis.

The perturbative eigenmodes of the plasma of gluons at finite temperature are the plasmons. In addition to the two transverse polarizations, there is also a longitudinal polarization, which is physical mode at finite temperature, obeying the Gauss law. Our approach is

to rewrite the theory as a theory of such plasmons. We work out interactions of the plasmons to the quadratic order in the coupling constant. The Coulomb interaction between plasmons shows the screened behaviour, as expected. Perturbative calculations with this Hamiltonian will be a HTL-resummed perturbation theory. This sets up the basic framework. The next step is to use this to calculate corrections to HTL-effects, some of which are under way.

# 2 Hamiltonian analysis

As mentioned in the introduction, our approach will be to simplify the Hamiltonian analysis. The action for a non-abelian gauge theory, with the HTL-effective action added, is given by [3]

$$\begin{aligned} S = -\frac{1}{4}\int F^2 \quad + \quad & \kappa \int d\Omega \Big[ d^2x^T S_{WZW}(G) + \frac{1}{\pi}\int d^4x \, \mathrm{Tr}(G^{-1}\partial_- G\, A_+ \\ & -A_-\partial_+ G G^{-1} + A_+ G^{-1} A_- G - A_+ A_-) \Big] \end{aligned} \tag{1}$$

where $S_{WZW}(G)$ is the standard Wess-Zumino-Witten (WZW) action for $G$. $G(x,Q)$ is a unitary matrix field depending on $x^\mu$ and on the unit vector $\vec{Q}$, $\vec{Q}\cdot\vec{Q} = Q^2 = 1$, and obeying $G^\dagger(x,\vec{Q}) = G(x,-\vec{Q})$. Also in (1), $\partial_\pm = \frac{1}{2}(\partial_0 \pm \vec{Q}\cdot\vec{\partial})$ and $A_\pm = \frac{1}{2}(A_0 \pm \vec{Q}\cdot\vec{A})$. The $G$-dependent term of the action includes integration over the angles of $\vec{Q}$. The parameter $\kappa$ is given in the lowest order calculation by $(N + \frac{1}{2}N_F)T^2/6$, where $N$ is the number of colors, $N_F$ is the number of quark flavors and $T$ is the temperature. For our purpose, $\kappa$ can be considered as a free parameter.

The Hamiltonian analysis of (1) has been given elsewhere [12]. With the gauge choice of $A_0 = 0$, the Hamiltonian is given by

$$H = \int d^3x \, \frac{E^2 + B^2}{2} \; + \; \frac{2\pi}{\kappa}\int d^3x \, d\Omega \; (J_+^2 + J_-^2) \tag{2}$$

where $E_i^a = \partial_0 A_i^a$ and $B_i^a = \epsilon_{ijk}(\partial_j A_k^a + \frac{1}{2} f^{abc} A_b^j A_c^k)$ are the usual nonabelian electric and magnetic fields. The currents $J_\pm$ are given by

$$\begin{aligned} J_+ &= \frac{\kappa}{4\pi} D_+ G\, G^{-1} = -it^a J_+^a \\ J_- &= -\frac{\kappa}{4\pi} G^{-1} D_- G = -it^a J_-^a \end{aligned} \tag{3}$$

and are related by $J_+(x,-\vec{Q}) = J_-(x,\vec{Q})$.

The equal-time commutator algebra which supplements the Hamiltonian (2) is given by

$$\begin{aligned}
[E_i^a(\vec{x}), A_j^b(\vec{y})] &= -i\delta^{ab}\delta_{ij}\delta(\vec{x}-\vec{y}) \\
[E_i^a(\vec{x}), J_+^b(\vec{y})] &= i\frac{\kappa}{4\pi}Q_i\delta^{ab}\delta(\vec{x}-\vec{y}) \\
[J_+^a(\vec{x},\vec{Q}), J_+^b(\vec{y},\vec{Q}')] &= -i\frac{\kappa}{4\pi}(Q\cdot\nabla_x\delta^{ab} - f^{abc}A^c(x))\delta(\vec{x}-\vec{y})\delta(Q,Q') \\
&\quad +if^{abc}J_+^c(\vec{x},\vec{Q})\delta(\vec{x}-\vec{y})\delta(Q,Q')
\end{aligned} \tag{4}$$

In addition to the Hamiltonian (2) and the operator algebra (4), we must also impose the Gauss law as a constraint selecting the physical states $|\psi\rangle$, i.e., we must require that $\mathcal{G}^a|\psi\rangle = 0$, where

$$\mathcal{G}^a = (\vec{D}\cdot E)^a + \int d\Omega\ (J_+^a + J_-^a) \tag{5}$$

Equations (2, 4, 5) define the theory. The rest of this paper will be devoted to the analysis of these equations.

The currents $J_\pm^a$ obey the generalized Kac-Moody algebra of (4). The first step in our approach will be to introduce a canonical set of variables $\phi^a(x,\vec{Q})$, $\Pi^b(x,\vec{Q})$ that depend on $\vec{Q}$ as well as $\vec{x}$, which obey the canonical algebra

$$[\phi^a(\vec{x},\vec{Q}), \Pi^b(\vec{y},\vec{Q}')] = -i\delta^{ab}\delta(\vec{x}-\vec{y})\delta(\vec{Q},\vec{Q}') \tag{6}$$

with all other commutators equal to zero.

We are interested in solving the theory to a certain order in the coupling constant or power of the structure constants $f^{abc}$, and so we can solve (4) as a series in $f^{abc}$. The solution for $J_+^a$ is given by

$$\begin{aligned}
J_+^a = &\quad \Pi^a - \frac{\kappa}{8\pi}Q\cdot\nabla\phi^a - \frac{\kappa}{4\pi}Q\cdot A^a \\
&- \frac{\kappa}{48\pi}f^{abc}\phi^b Q\cdot\nabla\phi^c + \frac{1}{2}f^{abc}\phi^b\Pi^c \\
&+ \frac{1}{24}f^{abc}f^{brs}(\Pi^r\phi^s\phi^c + \phi^c\Pi^r\phi^s) + \cdots
\end{aligned} \tag{7}$$

It is easily verified that this solves the algebra (4) to the quadratic order in $f^{abc}$. This also agrees with the expression (3) for $J_+^a$ in a suitable parametrization of $G(x,\vec{Q})$ in terms of $\phi^a(x,\vec{Q})$. $J_-^a$ is given by $\vec{Q}\to-\vec{Q}$ in (7). Since the unitary matrix $G(x,\vec{Q})$ obeys $G^\dagger(x,\vec{Q}) = G(x,-\vec{Q})$ we must have $\phi(x,-\vec{Q}) = -\phi(x,\vec{Q})$ and $\Pi(x,-\vec{Q}) = -\Pi(x,\vec{Q})$.

(Strictly speaking, this condition has to be imposed as a weak condition; however, as the comments in the concluding section make clear, this will not affect the reduction of the Hamiltonian which follows.) By direct substitution from (7) and keeping in mind the above mentioned property, we find the Hamiltonian to quadratic order in $f^{abc}$ as

$$
\begin{aligned}
\mathcal{H} \quad &= \int d^3x \Bigg[ \frac{E^2 + B^2}{2} + \frac{4\pi}{\kappa} \int_\Omega (\Pi - \frac{\kappa}{4\pi} Q \cdot A)^2 + \frac{\kappa}{16\pi} \int_\Omega (Q \cdot \nabla \phi)^2 \\
&\quad + \frac{1}{3} \int_\Omega f^{abc} (\Pi + \frac{\kappa}{8\pi} Q \cdot A)^a \phi^b Q \cdot \nabla \phi^c \\
&\quad + \frac{\pi}{3\kappa} \int_\Omega f^{abc} f^{ars} (\phi^b \Pi^c \phi^r \Pi^s) \\
&\quad + \frac{\kappa}{576\pi} \int_\Omega f^{abc} f^{ars} (\phi^b Q \cdot \nabla \phi^c)(\phi^r Q \cdot \nabla \phi^s) \\
&\quad - \frac{1}{12} \int_\Omega f^{abc} f^{ars} (\phi^b \Pi^c \phi^r Q \cdot A^s + \phi^r Q \cdot A^s \phi^b \Pi^c) + \cdots \Bigg]
\end{aligned} \tag{8}
$$

We must now consider the implementation of the Gauss law. This can also be done as a power series in $f^{abc}$. Notice that the Gauss law operator $\mathcal{G}^a$ has the form

$$
\begin{aligned}
\mathcal{G}^a &= \mathcal{G}^a_0 + \mathcal{G}^a_1 + \mathcal{G}^a_2 + \cdots \\
\mathcal{G}^a_0 &= \nabla \cdot E^a - \frac{\kappa}{4\pi} \int_\Omega Q \cdot \nabla \phi^a \\
\mathcal{G}^a_1 &= f^{abc} (A^b \cdot E^c + \int_\Omega \phi^b \Pi^c)
\end{aligned} \tag{9}
$$

$\mathcal{G}^a_0$ is of zero order in $f^{abc}$, $\mathcal{G}^a_1$ is of first order, etc. We have also used (7) to simplify (5). Our strategy for the Gauss law will be to find a unitary transformation $S = e^F$ which has the property of transforming $\mathcal{G}^a$ to $\mathcal{G}^a_0$. In other words we need

$$
S^{-1} \mathcal{G}^a S \approx \mathcal{G}^a_0 \tag{10}
$$

where the weak equality means "up to terms proportional to $\mathcal{G}^a_0$." Such a strategy seems to have been known to many people before; it has been used recently for Yang-Mills theories at zero temperature in [13]; a similar approach at finite temperature but without Debye mass was also used in [14] to clarify certain technical questions regarding the imaginary-time formalism. We first define a quantity

$$
\Delta^a = \nabla \cdot A^a + 2 \int_\Omega \frac{1}{(Q \cdot p)^2} Q \cdot \nabla \Pi^a \tag{11}
$$

$\Delta^a$may be considered as the gauge-fixing constraint conjugate to $\mathcal{G}_0^a$. Indeed we find

$$[\mathcal{G}_0^a(x), \Delta^b(y)] = -i\delta^{ab}(-\nabla_x^2 + m_D^2)\delta(x-y) \tag{12}$$

where $m_D = \sqrt{2\kappa}$ is the Debye mass. The inverse of the operator on the right hand side of (12) will be denoted by $G(x,y)$, i.e.,

$$G(x,y) = \int \frac{d^3p}{(2\pi)^3} \frac{e^{ip\cdot(x-y)}}{p^2 + m_D^2} \tag{13}$$

Defining

$$\chi^a(x) = \int d^3y\ G(y,x)\Delta^a(y) \tag{14}$$

we see that

$$[\mathcal{G}_0^a(x), \chi^b(y)] = -i\delta^{ab}\delta(x-y) \tag{15}$$

Writing $S = e^F \simeq 1 + F$, to the first nontrivial order, (10) reduces to

$$[\mathcal{G}_0^a(x), F] + \mathcal{G}_1^a \approx 0 \tag{16}$$

The solution to this equation is given by

$$\begin{aligned} F &= -i\int \chi^a f^{abc}(A^b\cdot E^c + \int_\Omega \phi^b\Pi^c) - \frac{i}{2}\int \chi^a\partial_i\chi^b f^{abc}\tilde{E}_i^c + \cdots \\ \tilde{E}_i^c &= E_i^c - \frac{\kappa}{4\pi}\int_\Omega Q_i\phi^a \end{aligned} \tag{17}$$

(Notice that $\nabla\cdot\tilde{E} = \mathcal{G}_0^a$.)

The physical states can now be easily constructed. Let $|\psi_0\rangle$ be any state obeying the condition $\mathcal{G}_0^a|\psi_0\rangle = 0$. Such states, as we shall see shortly, can be constructed by plasmon creation operators acting on the vacuum. Equation (10) then shows that physical states $|\psi\rangle$ may be written as

$$|\psi\rangle = S|\psi_0\rangle \tag{18}$$

In terms of such physical states we have for the matrix element of $\mathcal{H}$

$$\langle\psi_1|\mathcal{H}|\psi_2\rangle = \langle\psi_{1(0)}|S^{-1}\mathcal{H}S|\psi_{2(0)}\rangle \tag{19}$$

so that, in terms of the perturbatively constructed states $|\psi_0\rangle$, the Hamiltonian is effectively given by

$$\mathcal{H}_{eff} = S^{-1}\mathcal{H}S \tag{20}$$

$\mathcal{G}_0^a$ and $\Delta^a$ or $(\chi^a)$ will commute with the creation-annihilation operators for the plasmons and, after obtaining $\mathcal{H}_{eff}$, we can set $\mathcal{G}_0^a = 0$, $\Delta^a = 0$. Using the expression (8) for the Hamiltonian and $F$ as given by equation (17) we find

$$\mathcal{H}_{eff} = \mathcal{H}_0 + \mathcal{H}_{int} \tag{21}$$

with

$$\begin{aligned}\mathcal{H}_0 \;&=\; \int \frac{E^2 + (\epsilon_{ijk}\partial_j A_k)^2}{2} \\ &+\frac{4\pi}{\kappa}\int_\Omega (\Pi - \frac{\kappa}{4\pi} Q\cdot A)^2 + \frac{\kappa}{16\pi}\int_\Omega (Q\cdot\nabla\phi)^2\end{aligned} \tag{22}$$

and

$$\begin{aligned}\mathcal{H}_{int} \;&=\; \int f^{abc}\partial_i A_j^a A_i^b A_j^c + \frac{1}{4}\int f^{abc} f^{ars} A_i^b A_j^c A_i^r A_j^s \\ &+\frac{1}{3}\int f^{abc}(\Pi + \frac{\kappa}{8\pi} Q\cdot A)^a \phi^b Q\cdot\nabla\phi^c \\ &+\frac{2}{3}\int f^{abc}(\Pi + \frac{\kappa}{8\pi} Q\cdot A)^a \Big[\phi^b G(x,y)\tilde{\rho}^c(y) \\ &-\frac{1}{(Q\cdot p)^2} Q\cdot\nabla_x G(x,y)\tilde{\rho}^b(y) Q\cdot\nabla_x\phi^c(x)\Big] \\ &+\frac{\pi}{3\kappa}\int f^{abc} f^{ars}\phi^b\Pi^c\phi^r\Pi^s \\ &+\frac{\kappa}{576\pi}\int f^{abc} f^{ars}(\phi^b Q\cdot\nabla\phi^c)(\phi^r Q\cdot\nabla\phi^s) \\ &-\frac{1}{12}\int_\Omega f^{abc} f^{ars}(\phi^b\Pi^c\phi^r Q\cdot A^s + \phi^r Q\cdot A^s\phi^b\Pi^c) \\ &+\frac{1}{2}\int \tilde{\rho}^a(x) G(x,y)\tilde{\rho}^a(y) + \cdots\end{aligned} \tag{23}$$

where

$$\tilde{\rho}^a = f^{abc}(A^b\cdot E^c + \int_\Omega \phi^b\Pi^c) \tag{24}$$

The interaction part of the Hamiltonian now contains the screened Coulomb interaction between plasmons.

## 3 Plasmon operators

We now return to the construction of the plasmon operators which are eigenstates of the quadratic part of the Hamiltonian, viz., $\mathcal{H}_0$. Such operators must evidently obey the equation

$$[\mathcal{H}_0, \alpha_\lambda^{\dagger a}(p)] = \omega_\lambda(p)\, \alpha_\lambda^{\dagger a}(p) \tag{25}$$

so that many-plasmon states are obtained by multiple applications of $\alpha_\lambda^{\dagger a}(p)$ on the vacuum obeying the condition $\alpha_\lambda^a(p)|0\rangle = 0$. In (25), $\lambda$ is the polarization index. For a given spatial momentum $\vec{p}$, we define a triad of unit vectors $e_i^\lambda$, $\lambda = 1, 2, 3$, by

$$\begin{aligned} e_i^3 = \frac{p_i}{\sqrt{p^2}} \qquad & e^\lambda \cdot e^{\lambda'} = \delta^{\lambda\lambda'} \\ e_i^\lambda\, e_j^{\lambda'}\, \epsilon_{ijk} \quad &= \quad \epsilon^{\lambda\lambda'\alpha} e_k^\alpha \end{aligned} \tag{26}$$

One can, if desired, make an explicit choice

$$e_i^1 = \left( \frac{p_2}{\sqrt{p_1^2 + p_2^2}}, \frac{-p_1}{\sqrt{p_1^2 + p_2^2}}, 0 \right), \qquad e_i^2 = \frac{(p_3\, p_1, p_3\, p_2, -(p_1^2 + p_2^2))}{\sqrt{(p_1^2 + p_2^2)\, p^2}} \tag{27}$$

The plasmon creation operators may be taken to be of the form

$$\alpha_\lambda^{\dagger a}(p) = \int d^3x\; e^{-ip\cdot x} \left[ e^\lambda \cdot (\omega A^a - iE^a) + \int_\Omega (f_1 \Pi^a + f_2 \phi^a) \right] \tag{28}$$

Substituting this in (25) and simplifying, we see that solutions exist if $\omega_\lambda(p)$ are specific functions of $\vec{p}$ obeying certain dispersion relations. For the longitudinal plasmons, corresponding to $e_i^3$, we find that $\omega_L = \omega_3$ is given by

$$1 = \frac{\kappa}{2\pi} \int d\Omega \frac{(Q \cdot e^3)^2}{\omega_L^2 - (Q \cdot p)^2} \tag{29}$$

For the transverse polarizations we find

$$\omega_T^2 - p^2 = \omega_T^2\, \frac{\kappa}{2\pi} \int d\Omega \frac{(Q \cdot e^\lambda)^2}{\omega_T^2 - (Q \cdot p)^2} \tag{30}$$

The creation and annihilation operators, with the appropriate solutions for $f_1$, $f_2$ substituted into (28) are, for $\lambda = 1, 2, 3$,

$$\begin{aligned} \alpha_\lambda^{\dagger a}(p) &= \int d^3x \frac{N_\lambda(p) e^{-ip\cdot x}}{\sqrt{2\omega_\lambda V}} \left[ e^\lambda \cdot (\omega A^a - iE^a) + \int d\Omega\; f_\lambda(-\omega \Pi^a - \frac{i\kappa}{8\pi}(Q \cdot p)^2 \phi^a) \right] \\ \alpha_\lambda^a(p) &= \int d^3x \frac{N_\lambda(p) e^{ip\cdot x}}{\sqrt{2\omega_\lambda V}} \left[ e^\lambda \cdot (\omega A^a + iE^a) + \int d\Omega\; f_\lambda(-\omega \Pi^a + \frac{i\kappa}{8\pi}(Q \cdot p)^2 \phi^a) \right] \end{aligned} \tag{31}$$

where

$$
\begin{aligned}
f_\lambda &= \frac{2\ Q\cdot e^\lambda}{\omega_\lambda^2-(Q\cdot p)^2}\\
N_\lambda(p) &= \left[1+\frac{\kappa}{8\pi}\int d\Omega (Q\cdot p)^2 f_\lambda^2\right]^{-1/2}
\end{aligned}
\tag{32}
$$

We have used plane-wave normalization appropriate to a cubical box of volume $V=L^3$, so that $\vec{p}=(2\pi/L)\vec{n}$, $n_i\in\mathbf{Z}$. The operators $\alpha,\alpha^\dagger$ obey the expected commutator algebra

$$
[\alpha_\lambda^a(p),\alpha_{\lambda'}^{b\dagger}(p')]=\delta^{ab}\delta_{\lambda\lambda'}\delta_{\vec{p},\vec{p}'}
\tag{33}
$$

with all other commutators equal to zero. These operators create physical excitations in the sense that

$$
[\mathcal{G}_0^a(x),\alpha_\lambda^{b\dagger}(p)]=0
\tag{34}
$$

The operators $S^{-1}\alpha_\lambda^{a\dagger}(p)S$ are then operators which commute with Gauss law. We can also verify that $\Delta^a(x)$ commutes with $\alpha,\ \alpha^\dagger$. Thus for matrix elements with states built up of $\alpha^\dagger$'s, it is consistent to set $\mathcal{G}_0^a=0,\ \Delta^a=0$.

We now turn to the question of expressing the Hamiltonian (23) in terms of the plasmon operators. The plasmons are collective mode excitations in the plasma. In general, one can write every operator, such as $A,E,\phi,\Pi$, in terms of $\alpha,\ \alpha^\dagger$ plus operators $\beta,\ \beta^\dagger$ which represent other type of excitations. If the nonplasmon excitations are weakly coupled to $\alpha,\alpha^\dagger$, then an effective theory for the plasmons can be obtained by keeping just the $\alpha,\alpha^\dagger$-terms in $A,E,\phi,\Pi$. This is, in general, how we could obtain an effective theory for any kind of collective excitations. However, if $\alpha,\alpha^\dagger$ form a complete set of operators, clearly this would not be an approximation, since there would be no independent $\beta,\beta^\dagger$-type modes anyway. As far as $A_i,\ E_i$ are concerned, it is easily seen that the plasmons do form a complete set. The operators $\phi(x,\vec{Q}),\ \Pi(x,\vec{Q})$ have, in principle, an infinity of fields defined just on spacetime; these may be viewed as the coefficients of the expansion of $\phi,\Pi$ in terms of spherical harmonics constructed from $\vec{Q}$. Thus completeness in terms of $\alpha,\ \alpha^\dagger$ may seem doubtful. However, most of the degrees of freedom corresponding to the arbitrariness of the $\vec{Q}$-dependence are irrelevant to us. This is easily seen by going back to the action (1) and eliminating $G$ by its equation of motion. As shown elsewhere, there are no new propagating degrees of freedom in $G$. In fact, the elimination of $G$ leads to a purely $A$-dependent action and the analysis of this action shows that the only physical modes are the plasmons. (The action, with $G$ eliminated, is nonlocal in time making a Hamiltonian analysis difficult; this is why we need the auxiliary field.) We may therefore use the plasmons as the only modes

relevant to the interactions and reduce $\mathcal{H}$ entirely in terms of $\alpha, \alpha^\dagger$. First of all, we write $A_i^a(x)$ as

$$A_i^a(x) = \sum_{\lambda,p} \left[ a_{\lambda i} \alpha_\lambda^a(p) + a_{\lambda i}^* \alpha_\lambda^{a\dagger}(p) \right] \tag{35}$$

Then, from (33), we find $a_\lambda(p) = -[\alpha_\lambda^{a\dagger}, A_i^a(x)]$ and using (31),

$$a_{\lambda i}(p) = \frac{N_\lambda e^{-ip\cdot x}}{\sqrt{2\omega_\lambda V}}\ e_i^\lambda \tag{36}$$

We thus have

$$A_i^a(x) = \sum_{\lambda,p} \frac{N_\lambda(p)}{\sqrt{2\omega_\lambda V}} \left[ \alpha_\lambda^a(p) e_i^\lambda e^{-ip\cdot x} + \alpha_\lambda^{a\dagger}(p) e_i^\lambda e^{ip\cdot x} \right] \tag{37}$$

In a similar way we find

$$E_i^a(x) = \sum_{\lambda,p} \frac{N_\lambda(p)}{\sqrt{2\omega_\lambda V}}(-i\omega_\lambda) \left[ \alpha_\lambda^a(p) e_i^\lambda e^{-ip\cdot x} - \alpha_\lambda^{a\dagger}(p) e_i^\lambda e^{ip\cdot x} \right] \tag{38}$$

$$\phi^a(x) = \int d^3y\ K_i(x,y) E_i^a(y) \tag{39}$$

$$\Pi^a = \frac{\kappa}{8\pi} \int d^3y\ (\vec{Q}\cdot\nabla)^2 K_i(x,y) A_i^a(y) \tag{40}$$

where

$$K_i(x,y,\vec{Q}) = \int \frac{d^3p}{(2\pi)^3} e^{ip\cdot(x-y)} \sum_\lambda \frac{2\vec{Q}\cdot e^\lambda e_i^\lambda}{\omega_\lambda^2 - (\vec{Q}\cdot\vec{p})^2} \tag{41}$$

Expressions (37- 40) are easily checked to be compatible with the conditions $\mathcal{G}_0^a = 0,\ \Delta^a = 0$. The Hamiltonian (23) can now be expanded in terms of $\alpha,\ \alpha^\dagger$ in a completely straightforward way; we just have to substitute the mode expansions (37-40) in (23).

# 4 Discussion

We shall conclude with some comments on the treatment of the auxiliary variables $\phi, \Pi$. The conditions $\Pi(x, -\vec{Q}) = -\Pi(x, \vec{Q})$ and $\phi(x, -\vec{Q}) = -\phi(x, \vec{Q})$ have to be imposed as weak conditions. Otherwise one may get inconsistencies in the application of the current commutation rules. Consider the expansion of the fields into harmonics on the two-sphere

given by $\vec{Q}$. We may write

$$\begin{aligned} \phi(x,\vec{Q}) &= \sum_{lm} \phi_{lm}(x)\; Y_{lm}(\vec{Q}) \\ \Pi(x,\vec{Q}) &= \sum_{lm} \Pi_{lm}(x)\; Y_{lm}(\vec{Q}) \end{aligned} \tag{42}$$

The canonical commutation rules show that $[\phi_{lm}(x), \Pi_{l'm'}(x')] = i\delta_{ll'}\delta_{mm'}\delta(x-x')$. The requirement of these fields being odd under $\vec{Q} \to -\vec{Q}$ is equivalent to $\Pi_{lm} \approx 0,\ \phi_{lm} \approx 0$ for even values of $l$. $\Pi_{lm} \approx 0$ for even $l$ are first class constraints in the parlance of constraint analysis and the conditions $\phi_{lm} \approx 0$ may be taken as the gauge-fixing constraints for them. One may set them to zero strongly one we have redefined commutators via Dirac brackets. Since the other fields $\chi = (A_i, E_i, \phi_{lm}, \Pi_{lm})$ (for odd $l$) commute with the constraints, the Dirac brackets are the same as the Poisson brackets. Thus we may set the constraints to zero at this stage.

In the Hamiltonian, the quadratic terms have no mixing of the even $l$- and odd $l$-components of the fields. In the interaction terms, there is mixing but we see that commutators of $\chi$'s always generate terms involving the constraints and hence it is consistent to set the constraints to zero in the expression for the Hamiltonian.

In summary, we have expressed the Hamiltonian with the HTL effects added in terms of plasmon creation and annihilation operators. One can use this for calculations of different types of higher order corrections; one can also use this as a starting point for variational calculations involving the thermal distribution of plasmons. Some of these are currently under way.

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